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# Analysis of a Thin Optical Lens Model

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**Abstract**

In this article a thin optical lens model is considered. It is shown that the limits of its applicability are determined not only by the ratio between the thickness of the lens and the modules of the radii of curvature, but above all its geometric type. We have derived the analytical criteria for the applicability of the model for different types of lenses. Quantitative estimations were made for practically important cases of a glass lens in air and air lens in water.

**Keywords**: A thin optical lens model, he limits of the model's applicability,

## Introduction

One of the major classes of concepts considered in school and university physics courses are physical idealization – the objects not existing and unrealizable in the real world, but having their pre-images in it. The ideal models, expressed in an appropriate form of sign, become abstract mathematical models, which allow ones to explore them on a quantitative level and make the interpretation of the obtained results.

An important characteristic of such models is the simulation interval. It refers to a system of conditions within which the identification of the object with the model is achieved. For example, the classical Newton's second law describes the mechanical properties of the particle, if its speed is much less than the light's speed.

It should be noted that represented in the form of inequalities $a << b$, the simulation interval is quite abstract and "opaque". To overcome this limitation one should calculate relative error $\varepsilon$, which arises when we neglect the quantity of $a$ in the expression describing the phenomenon under consideration. It is sufficient to specify the range of values $a$ for which the error does not exceed the value dictated by the required accuracy for finding the investigated characteristic (as a rule, $\varepsilon_{\max} = 5\%$). This process may be called a quantitative estimation of the physical idealization applicability limits.

## Calculations for a thin optical lens model

As an example let us consider one of the most general idealizations in geometrical optics – a thin optical lens model. The latter refers to the transparent for this interval of electromagnetic waves region of space bounded by two refracting surfaces having a common axle or two mutually perpendicular planes of symmetry, and for which takes place the following condition:

$$t << \left|R_{1,2}\right| \ (R_{1,2} \neq 0). \qquad (1)$$

Here $t > 0$ – "axis" lens thickness that is the distance between the poles of its surfaces; $R_{1,2}$ is the radius of the lens curvature measured along its symmetry axis. Further we will specify this criterion and investigate it on quantitative level.

In the paraxial approximation, with consideration of finite value $t$, the generalized formula of lens has following form:

$$\frac{1}{f}-\frac{1}{d}=D=\frac{n-1}{nR_1R_2}\left[n(R_2-R_1)+(n-1)t\right], \tag{2}$$

where $f$, $d$ respectively are the distances from the second and the first principal planes to

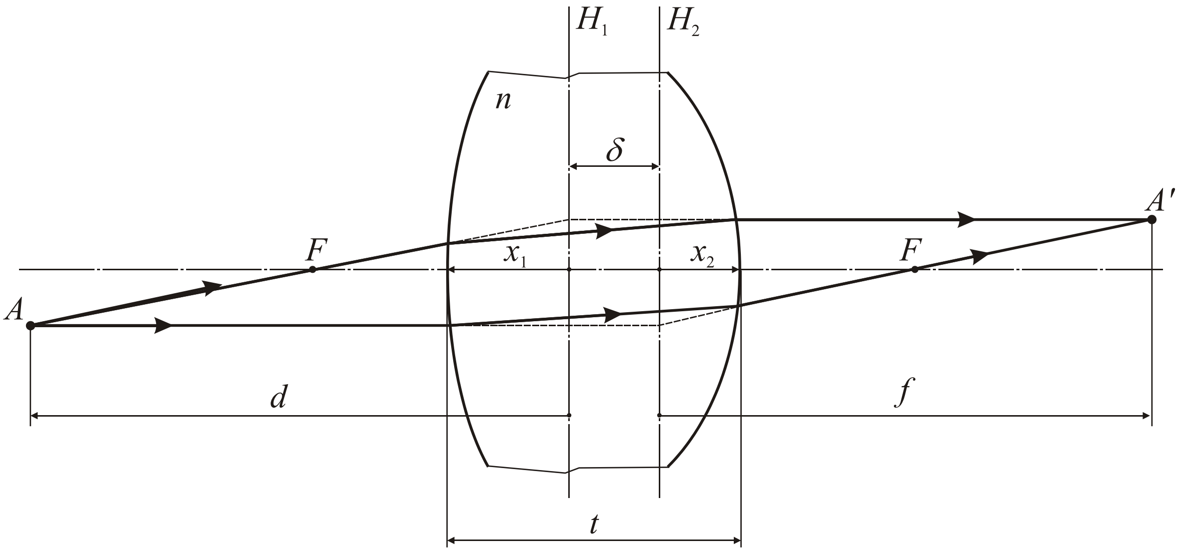


Fig. 1. Image formation of the luminous point $A$ in the thick lens

the image and the luminous point $A$ (Fig. 1). The focal lengths are also measured from the principal planes.

The right-hand side of expression (2) determines the effective optical power $D$ of a thick lens and is called the Gullstrand's equation (thick lens formula or Lensmaker's equation) [1]. This equation is valid for the case when both sides of the lens are situated in the same environment (practically important exceptions from this case are the human eye and oil-immersion microscope lens). In this case $n>0$ is the refractive index of the lens material with respect to the environment in which it is located.

The coordinates of the poles relative to the lens of principal planes can be calculated with the formulas [2]: $x_1=tR_1/\left[n(R_2-R_1)+(n-1)t\right]$, $x_2=tR_2/\left[n(R_2-R_1)+(n-1)t\right]$. Then the distance between the principal planes is

$$\delta=t\left|\frac{(n-1)(R_2-R_1+t)}{n(R_2-R_1)+(n-1)t}\right|. \tag{3}$$

Therefore, the relation (1) will be the thin lens formula under the conditions: $\varepsilon_D=\left|(D-D_0)/D\right|<<1$, $r=\delta\left|D\right|<<1$. With regard to (2), (3), these conditions take the form:

$$\begin{cases} \varepsilon_D \approx \dfrac{t}{n}\left|\dfrac{n-1}{R_2-R_1}\right| << 1 \\ r \approx \dfrac{(n-1)^2 t}{n}\left|\dfrac{R_2-R_1}{R_1R_2}\right| << 1 \end{cases} . \tag{4}$$

For each value of $n$ the system (4) allocates on the plane $(R_1/t,\, R_2/t)$ region for which the thin lens approximation is valid. On Fig. 2 (a, b) the results of numerical calculations of the level curves for $n = 1{,}6$ (the average value of the refractive index of glass) and $n = 0{,}75$ (air lens in water), along which the values $\varepsilon_D$ and $r$ are equal to 5%, are presented. Regions that correspond to large values of these quantities are shown in gray.

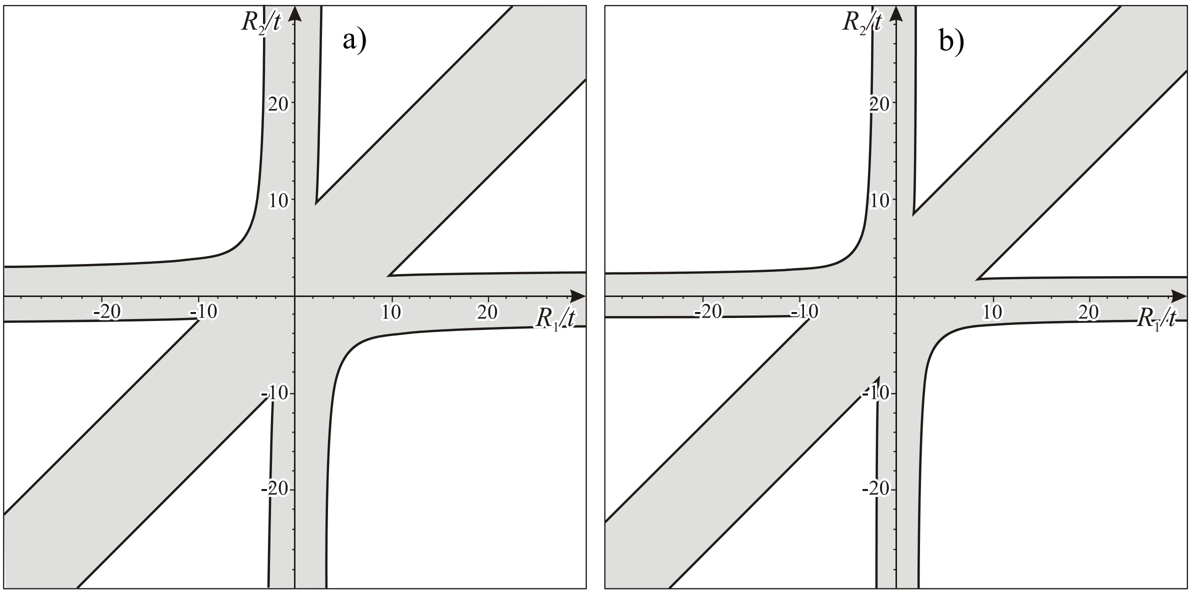


Fig. 2. The correctness (white) and incorrectness (gray) regions of thin lens approximation for $\varepsilon_D = r = 5\%$. a) $n = 1{,}6$; b) $n = 0{,}75$

Analysis of these diagrams shows that the limits of a thin lens applicability model are determined, above all, by its geometric type. Thus, when the curvature radii of the lens have different signs (biconvex and biconcave lenses), the latter may be considered as thin if each of modules is more than 2-4 times greater than its thickness (asymptotes of curves in these cases are the lines $|R_{1,2}| = (n-1)^2 t/nr$). If $\mathrm{sgn}(R_1) = \mathrm{sgn}(R_2)$ (positive and negative meniscus), one of the radii should satisfy the condition:

$$\left|R_{1,2}\right| > \frac{(n-1)^2 t}{nr} \tag{5}$$

(for the values $n$, $r$, described above, $\left|R_{1,2}\right| > (2-4)t$), and another – the condition:

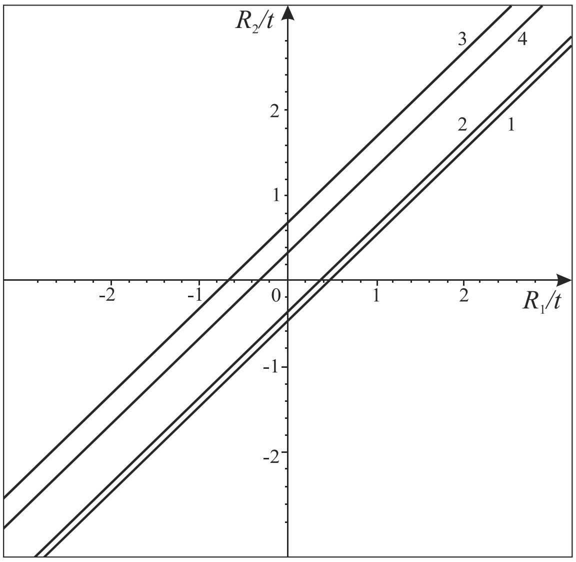


Fig. 3. The lines on the plane $(R_1/t,\, R_2/t)$, along which the effective optical power is zero. 1) $n = 1{,}9$; 1) $n = 1{,}6$;1) $n = 0{,}75$;1) $n = 0{,}6$

$$\left|R_{2,1}\right| > \frac{(n-1)t}{nr}\left[\frac{(n-1)\varepsilon_D + r}{\varepsilon_D r}\right] \tag{6}$$

(for the values $n$, $r$, $\varepsilon_D$, described above, $\left|R_{2,1}\right| > (8-10)t$). At the same time

$$\left|R_2 - R_1\right| > \frac{t\left|n-1\right|}{n\varepsilon_D} \tag{7}$$

(for the values $n$, $r$, $\varepsilon_D$, described above, $\left|R_2 - R_1\right| > (6-8)t$).

More stringent requirements for the lenses in the form of a meniscus are due to the fact that with $R_1 \to R_2$ their optical power will strong depend on the thickness. Thus, when $t = 0$ they will appear as ordinary blanket lenses with $D = 0$.

Gullstrand's equation also allows for finding the relationship between $R_{1,2}$ and $t$, in which the image becomes a telescopic one ( $D = 0$ ).From the expression (2) follows that this equation has the form of line $R_1/t - R_2/t = (n-1)/n$ on the plane $(R_1/t,\, R_2/t)$. Fig. 3 shows a set of such lines, constructed for several values of the refraction index. It's seen that for $n > 1$ ( $n < 1$ ) the effective optical power can not vanish for biconcave (biconvex) lenses for arbitrary pairs of the curvature radii values. The lenses in the form of the meniscus have no such limits.

It should be noted that the sign of the effective optical power can characterize the optical properties of the lens (converging, diverging) only when it is thin. In this case $D$ is simply called the optical power With $t \neq 0$ sign of $D$ may not always indicate the type of focal point of the lens (real or imaginary). This statement is easily illustrated by the biconvex lens with $n > 1$. When the thickness of the lens is small, its focal point is situated on the right of the second pole (Fig. 4a). With $t = t_x = nR_1/(n-1)$ refracted by the first surface rays intersect in the second pole (Fig. 4b). If $t > t_x$ the output beam becomes divergent (Fig. 4c),

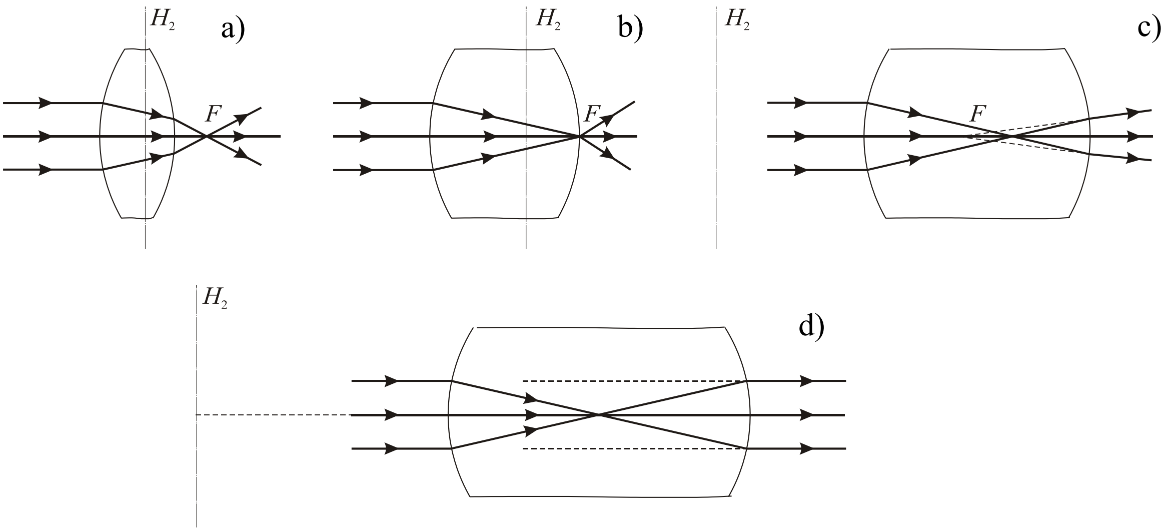


Fig. 4. Position of the back focal point in a thick biconvex lens with different values of its axial thickness. a) $t < t_x$; b) $t = t_x$; c) $t_x < t < t_0$; d) $t = t_0$

while $D$ remains positive. Finally, with $t = t_0 = n(R_1 - R_2)/(n-1)$ the output beam is converted into parallel (Fig. 4 d) and the effective optical power becomes equal to zero.

**Conclusions**

The applicability limits of a thin lens model are determined not only by the ratio between the thickness of the lens and the curvature radii modules, but above all by its geometric type. In the case of biconvex and biconcave lenses the criteria for applicability of the model are defined by the condition (5), while for the meniscus – by conditions (5-7). A significant thickness of the lens can lead to the appearing of a telescoping effect, as well as to discrepancy between the lens effective power sign and the lens optical type.

**References**

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